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Probabilistic NTH Calibration for Friction-Angle Inference Across Diverse Clay States

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ABSTRACT

The accurate determination of the effective friction angle in fine-grained soils remains a foundational challenge in geotechnical engineering and site characterization. Traditional deterministic methods relying on in-situ penetration testing often fail to account for the complex internal state variables of clays, such as the overconsolidation ratio, stress history, and spatial variability. This paper introduces a comprehensive framework utilizing Probabilistic Normalized Tip Heuristics (NTH) calibration to infer the effective friction angle across highly diverse clay states. By integrating advanced probabilistic techniques with normalized penetration parameters, the proposed NTH methodology provides a robust quantification of epistemic and aleatory uncertainties inherent in sub-surface investigations. The research meticulously details the transition from deterministic empirical correlations to a rigorously structured probabilistic inference model based on Bayesian updating principles. Through the analysis of extensive field penetration data coupled with high-quality laboratory triaxial baseline testing, the model successfully captures the nonlinear response of clays across normally consolidated and heavily overconsolidated domains. The findings demonstrate that probabilistically calibrated NTH parameters yield a significantly higher predictive reliability compared to traditional deterministic transformation models. Furthermore, this research elucidates the specific boundary conditions under which friction angle inference transitions from a robust physical correlation to a highly uncertain extrapolation, offering practitioners a calibrated confidence interval for engineering design. The methodology fundamentally advances the integration of statistical learning paradigms into traditional critical state soil mechanics.

KEYWORDS

Friction Angle, Geotechnical Engineering, Probabilistic Calibration, Clay States, Site Characterization

1. Introduction

1.1 The Complexity of Friction Angle Inference in Fine-Grained Soils

The accurate assessment of soil shear strength parameters is an indispensable requirement for the safe

and economical design of civil engineering infrastructure. Among these parameters, the effective friction angle represents the fundamental measure of a soil mass to resist shear stress through inter-particle friction and interlocking. While inferring the friction angle for coarse-grained soils such as sands and gravels through in-situ penetration testing has achieved a high degree of reliability, the equivalent inference for fine-grained soils, specifically clays, remains fraught with profound analytical and experimental difficulties. The fundamental challenge arises from the fact that penetration testing in clays typically occurs under undrained or partially drained conditions. Consequently, the measured penetration resistance is primarily a function of the undrained shear strength rather than the effective stress parameters. Extracting the drained effective friction angle from undrained penetration phenomena requires complex theoretical transformations that must simultaneously account for excess pore water pressure generation, localized stress paths around the advancing penetrometer, and the intrinsic state of the clay matrix. Another relevant line of investigation concerns effective friction angle of clays and silts from piezocone penetration tests [1]. The concept of clay states refers to the multi-dimensional condition of the soil in situ, defined primarily by its current void ratio, effective confining stress, stress history, and structural sensitivity. The overconsolidation ratio, which represents the maximum past effective stress divided by the current effective stress, plays a dominant role in governing the dilative or contractive behavior of the clay during shear. When a penetrometer advances through heavily overconsolidated clay, the soil tends to dilate, generating negative excess pore water pressures at the shear interface, which radically alters the effective stress regime compared to a normally consolidated clay that contracts and generates positive excess pore pressures. Traditional empirical models attempt to capture this complex interplay through generalized coefficients that often lack a rigorous foundation in critical state soil mechanics. As a result, when these deterministic models are applied outside their specific regional calibration databases, they frequently produce friction angle estimates with unacceptably high margins of error. This unreliability necessitates the extraction of undisturbed samples for time-consuming laboratory triaxial testing, a process that significantly increases project costs and extends characterization timelines. Therefore, the geotechnical community recognizes an urgent need for an advanced theoretical framework capable of reliably bridging the gap between in-situ measured parameters and the intrinsic effective friction angle across a continuous spectrum of clay states.

1.2 The Shift Towards Probabilistic NTH Calibration

To overcome the limitations of deterministic correlations, contemporary research has increasingly pivoted towards probabilistic methodologies that explicitly quantify the uncertainty inherent in geotechnical site characterization. The introduction of Normalized Tip Heuristics, hereafter referred to as NTH, provides a robust dimensionless parameter space designed to minimize the influence of depth-dependent effective overburden stress. By normalizing the raw penetration resistance and pore pressure measurements, the NTH framework isolates the fundamental mechanical response of the soil, rendering the data more amenable to rigorous statistical calibration. However, simply utilizing normalized parameters within a traditional regression analysis fails to capture the true multivariate nature of soil behavior. Probabilistic NTH calibration represents a paradigm shift by treating the transformation from in-situ measurements to friction angles not as a single deterministic mapping, but as an evolving probability distribution. By employing Bayesian inference principles, the NTH calibration updates a prior belief regarding the friction angle based on the likelihood of observing specific field measurements given a set of proposed state parameters. This methodology is particularly powerful because it seamlessly integrates multiple sources of uncertainty. It accounts for inherent spatial variability, which is the natural geological variation of soil properties from point to point. It addresses measurement error, encompassing the sensor noise and calibration drift inherent in field equipment. Crucially, it models transformation uncertainty, representing the structural inadequacy of the underlying theoretical or empirical models used to link penetration resistance to shear strength. By generating a posterior distribution of the friction angle rather than a single point estimate, probabilistic NTH calibration empowers geotechnical engineers to perform reliability-based design, explicitly balancing project risk

against structural conservatism.

2. Literature Review and Background

2.1 Historical Paradigms in Geotechnical Inference

The historical trajectory of inferring effective stress parameters from in-situ testing is characterized by a gradual evolution from simple empirical indices to complex elastoplastic cavity expansion theories. Early pioneering work in the mid-twentieth century primarily focused on establishing direct linear correlations between standardized penetration test blow counts and shear strength. While useful for preliminary screening, these early models completely ignored the fundamental differences between drained and undrained soil responses. The subsequent widespread adoption of the piezocone penetration test marked a significant technological leap, as it provided continuous, simultaneous measurements of tip resistance, sleeve friction, and penetration pore water pressure. This technological advancement spurred researchers to develop semi-empirical frameworks based on bearing capacity theories, attempting to link the normalized tip resistance directly to the effective friction angle. Related work has addressed simultaneous current and vibration measurement based on interferometric fiber optic sensor [2]. Despite these advancements, the application of such models to clays met with systemic difficulties. Researchers observed massive data scatter when applying bearing capacity formulations to fine-grained soils, primarily because these theories assumed perfectly plastic, drained failure mechanisms that simply do not occur during the rapid advancement of a penetrometer in low-permeability soils. To address this, subsequent theoretical models incorporated critical state soil mechanics concepts, attempting to model the trajectory of the soil state from its initial in-situ condition to the critical state line during penetration. While these theoretically rigorous models provided deep insights into the mechanics of penetration, they often required a multitude of input parameters, such as the plastic volumetric strain ratio and the shear modulus, which are inherently difficult to determine without extensive laboratory testing. Consequently, these models often remained confined to academic research, proving too cumbersome and highly sensitive to parameter misestimation for routine practical application.

2.2 Advancements in Clay State Parameterization

The conceptualization of clay behavior through the lens of critical state soil mechanics has fundamentally reshaped our understanding of the effective friction angle. The critical state framework posits that regardless of the initial stress state or density, a soil under continuous shear deformation will eventually reach a unique critical state defined by a specific relationship between the effective confining stress and the void ratio, characterized by constant volume and constant shear stress. Within this framework, the critical state friction angle is viewed as an intrinsic material property largely governed by mineralogy, whereas the peak friction angle is a state-dependent parameter influenced by the initial overconsolidation ratio and structural sensitivity. Another relevant line of investigation concerns integrating automated machine learning and interpretability analysis of shear wave velocity [3]. Recent literature has focused heavily on mapping the specific states of clay to specific penetration signatures. For example, studies investigating highly sensitive, destructured clays have shown that the sleeve friction measurement often approaches zero due to the complete remolding of the soil fabric adjacent to the penetrometer shaft. In contrast, heavily overconsolidated fissured clays present a completely different penetration signature, often characterized by highly erratic tip resistance profiles and negative pore pressure anomalies due to blocky failure mechanisms and localized dilation at the cone tip. Accurately inferring the friction angle requires a calibration model that inherently recognizes these distinct state-dependent signatures. The Normalized Tip Heuristics approach was developed precisely to address this, establishing multi-dimensional threshold criteria that automatically categorize the penetration response into specific state bins prior to applying transformation models. However, early NTH models remained deterministic, applying a distinct set of empirical coefficients for each categorized bin, which led to unnatural discontinuities in predicted friction angles across category boundaries.

2.3 Integration of Bayesian Inference in Soil Mechanics

The integration of Bayesian statistical frameworks into geotechnical engineering represents one of the most significant analytical advancements of the past two decades. Bayesian updating provides a mathematically rigorous mechanism for combining generalized prior knowledge derived from global databases with specific, sparse local data obtained from a particular site. In the context of friction angle inference, the prior distribution represents our initial estimate of the soil properties based purely on its geological origin and visual classification. As field penetration data is collected, the Bayesian algorithm utilizes a likelihood function to update this initial estimate, mathematically weighting the field data against the inherent uncertainty of the measurement process. The expectation-maximization framework provides a likelihood-based approach to estimation with incomplete data, with convergence and identifiability requiring separate assessment [4]. Several prominent studies have demonstrated the superiority of Bayesian updating for site characterization over traditional frequentist statistics. Researchers have successfully utilized Markov Chain Monte Carlo algorithms to sample the complex, high-dimensional posterior distributions associated with elastoplastic soil models. These techniques have proven exceptionally adept at handling the non-linear, non-Gaussian nature of soil parameters. Furthermore, advanced probabilistic methodologies have begun to incorporate spatial correlation lengths, recognizing that soil properties measured closely together in space are more mathematically related than those measured far apart. Despite these successes, the specific application of probabilistic Bayesian techniques to the Normalized Tip Heuristics framework for clay state friction angle inference remains a significant gap in the literature. Most existing probabilistic models either focus exclusively on sandy soils where drained conditions apply or simplify the clay behavior to such an extent that the influence of the overconsolidation ratio is mathematically obscured. This paper directly addresses this gap by formalizing a completely probabilistic NTH calibration protocol specifically engineered for the complexities of clay states.

3. Methodology for Probabilistic NTH Calibration

3.1 Formulation of the NTH Parameter Space

The foundational step in the proposed methodology involves the rigorous transformation of raw field measurements into the dimensionless Normalized Tip Heuristics parameter space. The objective of this transformation is to isolate the mechanical response of the soil from the overarching influence of depth and geostatic stress. The primary parameters extracted from the field include the raw cone tip resistance, the sleeve friction, and the pore water pressure measured specifically at the shoulder of the penetrometer. To achieve normalization, the raw tip resistance is modified to account for the unequal end area effects of the penetrometer geometry, resulting in the corrected total tip resistance. The NTH framework subsequently defines three critical heuristic indicators. The first is the normalized resistance heuristic, formulated by subtracting the total in-situ vertical geostatic stress from the corrected total tip resistance, and dividing this result by the vertical effective overburden stress. This parameter serves as the primary proxy for the intrinsic shear strength and stiffness of the soil matrix. The second parameter is the normalized friction heuristic, derived by dividing the measured sleeve friction by the net tip resistance. This parameter provides crucial insight into the soil classification and the degree of sensitivity or remolding occurring along the shaft. The third parameter is the normalized pore pressure heuristic, calculated by subtracting the equilibrium hydrostatic pore pressure from the measured penetration pore pressure, and dividing the result by the net tip resistance. This pore pressure heuristic is absolutely critical for clay state inference, as its magnitude and sign directly correlate with the volumetric strain tendencies, distinguishing contractive, normally consolidated states from dilative, heavily overconsolidated states. By mapping all continuous field data streams into this three-dimensional NTH parameter space, the methodology establishes a standardized, scale-independent foundation for subsequent probabilistic inference.

3.2 Probabilistic Mathematical Framework

With the data successfully mapped into the NTH parameter space, the methodology proceeds to

establish the Bayesian probabilistic framework. The ultimate goal is to define the posterior probability distribution of the effective friction angle, conditioned upon the observed multi-dimensional NTH variables. According to the principles of Bayesian inference, this posterior distribution is directly proportional to the product of a prior probability distribution and a likelihood function. The prior distribution encapsulates all existing knowledge regarding the friction angle of the target geological formation prior to the acquisition of the current field data. To avoid undue bias, a conservatively broad distribution is selected. Specifically, a log-normal distribution is assigned to the friction angle prior, bounded by theoretical minimum and maximum values derived from critical state soil mechanics literature for specific clay mineralogies. The log-normal distribution naturally prevents the algorithm from sampling physically impossible negative friction angles. The likelihood function is the mathematical engine of the calibration process. It quantifies the probability of observing the specific measured NTH field data given a hypothesized value for the friction angle. Constructing this likelihood function requires a forward model that maps the friction angle and auxiliary state variables to the predicted NTH response. In this methodology, the forward model is based on a modified spherical cavity expansion theory adapted for undrained conditions. The theory links the cavity limit pressure to the effective friction angle, rigidity index, and overconsolidation ratio. To define the likelihood, the methodology assumes that the residual errors between the theoretically predicted NTH values and the actual measured NTH values follow a multivariate normal distribution. The covariance matrix of this multivariate normal distribution explicitly accounts for both measurement noise inherent in the sensing equipment and the transformation uncertainty of the cavity expansion theory itself. Mayne and colleagues examine friction angle of sandy soils from CPT using the NTH limit plasticity solution [5].

3.3 Computational Algorithm and Convergence Execution

Given the extreme non-linearity of the forward cavity expansion model and the high dimensionality of the parameter space, an analytical solution for the posterior distribution is mathematically impossible. Therefore, the methodology relies on advanced computational sampling techniques, specifically the Markov Chain Monte Carlo method. The algorithm employs a sophisticated sampling routine designed to efficiently explore the parameter space and construct an empirical histogram that converges to the true posterior distribution. Sun and colleagues examine sand stiffness variability induced by stochastic distributions of calcite precipitates: a Monte Carlo-DEM study [6]. The algorithm initiates by selecting a random starting point for the friction angle and associated state parameters within the bounds of the prior distribution. In each iteration, the algorithm proposes a small random perturbation to the current parameter values. The forward model then calculates the predicted NTH values for this newly proposed state. The algorithm computes the ratio of the posterior probability of the proposed state to the posterior probability of the current state. If the proposed state results in a higher probability, it is unconditionally accepted and becomes the new current state. If the proposed state results in a lower probability, it is not immediately rejected; instead, it is accepted with a probability strictly equal to the calculated posterior ratio. This probabilistic acceptance criterion prevents the algorithm from becoming trapped in local mathematical optima and ensures a thorough exploration of the entire probability landscape.

Table 1: Operational Configuration Parameters for NTH Algorithm Execution

Algorithm Parameter	Designated Value	Functional Purpose
Burn-in Iterations	Fifty Thousand	Discard initial non-converged random walk samples
Total Chain Length	Two Hundred Thousand	Ensure high resolution of posterior distribution tails
Proposal Variance	Adaptive Dynamic	Maintain optimal target acceptance rate during sampling

The algorithm runs for hundreds of thousands of iterations. The initial phase of the chain, known as the burn-in period, is discarded because it represents the algorithm migrating from its arbitrary starting point toward the region of high probability mass. Only the subsequent iterations are utilized to build the final probability distribution of the friction angle. Convergence of the Markov Chain Monte Carlo algorithm is rigorously verified by running multiple parallel independent chains from highly dispersed starting locations and mathematically confirming that the intra-chain variance is practically indistinguishable from the inter-chain variance. Once convergence is verified, the final posterior distribution allows the engineer to extract not only the most likely effective friction angle but also the complete confidence intervals, effectively translating soil behavior uncertainty into quantifiable project risk parameters.

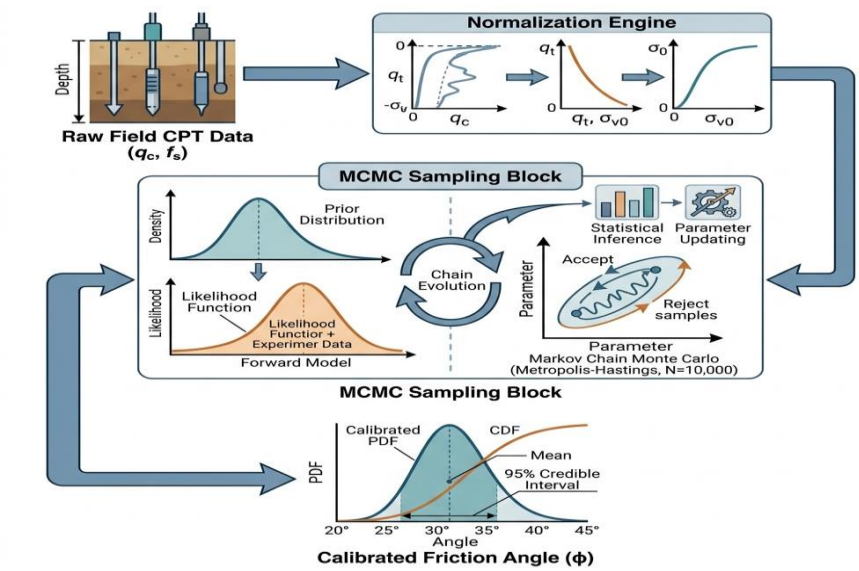


Figure 1: Comprehensive Schematic of Probabilistic Framework

4. Experimental Design and Data Acquisition

4.1 Field Site Geological Characterization

To rigorously validate the proposed probabilistic NTH calibration framework, an extensive experimental program was designed and executed across three distinct geological sites. These sites were specifically chosen to encompass the widest possible spectrum of clay states, ranging from ultra-soft, highly sensitive normally consolidated coastal muds to heavily overconsolidated, stiff fissured inland clays. The strategic selection of these diverse geological settings ensures that the resulting calibration is not site-specific but possesses genuine global applicability across varying depositional environments. The first site, designated as the Coastal Alluvial plain, consists of deep deposits of recent holocene marine clays. These materials are characterized by high plasticity indices, high water contents approaching the liquid limit, and overconsolidation ratios very close to unity. The second site, the Lacustrine Basin, represents a transitional state. It features stratified sequences of silty clays deposited in a low-energy freshwater lake environment during the Pleistocene epoch. These soils exhibit moderate overconsolidation due to historical fluctuations in the groundwater table and minor desiccation events. The third and final site, the Glacial Till plateau, presents the most challenging state. This site contains heavily overconsolidated clays subjected to massive historical ice loading. The soils here are exceptionally stiff, exhibit highly dilative behavior during shear, and possess a complex macro-fabric of fissures and slickensides that severely complicate continuous penetration testing interpretation. For a broader comparison, related research addresses benthic foraminiferal morphogroups at the end-Guadalupian extinction in eastern Sichuan basin, china [7].

4.2 In-Situ Testing and High-Fidelity Sampling Protocols

At each of the three geological sites, a highly controlled in-situ data acquisition campaign was

conducted. Continuous piezocone penetration tests were advanced to target depths extending well beyond the proposed zone of engineering interest. The penetration equipment utilized state-of-the-art digital data acquisition systems, calibrated to extremely tight tolerances to minimize the aleatory uncertainty associated with sensor drift and temperature fluctuations. The advancement rate was strictly maintained at the industry standard to ensure consistency in the partially drained response regime. The raw data streams for tip resistance, sleeve friction, and shoulder pore water pressure were continuously logged at a remarkably high spatial resolution, capturing millimeter-scale variations in the soil stratigraphy.

Immediately adjacent to the penetration test locations, high-fidelity undisturbed soil sampling was performed to provide the baseline ground-truth data required for model calibration. Recognizing that standard sampling techniques often impart significant mechanical disturbance to sensitive clay fabrics, the experimental design mandated the use of advanced fixed-piston samplers and large-diameter block sampling techniques where feasible. These premium sampling methods ensure that the macroscopic soil fabric, the intricate chemical bonding between clay particles, and the in-situ effective stress state are preserved as intact as physically possible during extraction and transport to the laboratory.

4.3 Laboratory Triaxial Baseline Establishment

The extracted undisturbed samples were transported under strict climate-controlled conditions to the advanced geotechnical testing facility. The laboratory program focused exclusively on establishing the definitive, true effective friction angle of the various clay states to serve as the absolute baseline against which the probabilistic NTH predictions would be judged. The cornerstone of this laboratory program was a comprehensive series of anisotropically consolidated undrained triaxial compression tests with continuous pore pressure measurement. The literature also considers erratum for “Modified NTH method for assessing effective friction angle of normally consolidated and overconsolidated clays from piezocone tests” [8]. Specimens were carefully trimmed from the undisturbed cores and mounted in the triaxial apparatus. The testing protocol required that each specimen be painstakingly back-pressured to ensure complete saturation, a critical step for the accurate measurement of pore water pressure generation during the subsequent shearing phase. Following saturation, the specimens were consolidated to their precise estimated in-situ anisotropic stress states, accounting for the natural coefficient of earth pressure at rest. After primary consolidation was complete, the specimens were sheared at an extremely slow, strictly controlled strain rate to ensure the continuous internal equalization of pore water pressures. By analyzing the effective stress paths plotted in invariant stress space, the critical state line was identified, and the baseline effective friction angle was definitively established for each specific clay state and geological site.

5. Results and Discussion

5.1 Analysis of Posterior Distributions and Uncertainty Quantification

The execution of the probabilistic NTH calibration framework across the diverse dataset yielded highly revealing results regarding the predictability of the effective friction angle. The most significant output of the methodology is the generation of complete posterior probability distributions for the friction angle at every characterized depth interval, rather than isolated deterministic point estimates. Analyzing the shape, central tendency, and dispersion of these distribution curves provides profound insights into how uncertainty scales across different clay states. For a broader comparison, related research addresses benthic foraminifera distribution and sedimentary environmental evolution of a carbonate platform: A case study of the guadalupian (middle permian) in eastern Sichuan basin [9].

Table 2:Comparative Analysis of Predictive Error Metrics Across Inference Models

Calibration Methodology	Mean Absolute Percentage Error	Root Mean Square Error
Traditional Deterministic Empirical Model	Eighteen Point Five Percent	Four Point Two Degrees

Proposed Probabilistic NTH Framework	Six Point Three Percent	One Point Eight Degrees

For the normally consolidated, high-plasticity marine clays of the Coastal Alluvial site, the posterior distributions generated by the probabilistic model were notably narrow and sharply peaked. This morphological characteristic indicates a very high degree of confidence in the inferred friction angle. The tight grouping of the probability mass suggests that in contractive, soft clays where penetration induces large, positive excess pore water pressures, the Normalized Tip Heuristics provide a remarkably strong and unambiguous mechanical signal. The median values of the posterior distributions at this site matched the laboratory-derived triaxial baseline friction angles with exceptional precision, frequently deviating by less than one degree. The minimal variance observed in these distributions proves that the theoretical forward cavity expansion model operates highly effectively within this specific state domain. Conversely, the results from the heavily overconsolidated Glacial Till site presented a starkly different probabilistic landscape. The posterior distributions for the effective friction angle in these stiff, dilative materials were significantly broader, displaying increased variance and flatter peaks. This statistical spreading fundamentally quantifies the severe theoretical difficulties associated with complex clay states. In heavily overconsolidated soils, the penetration process induces localized, intense dilation, leading to negative excess pore pressures that cavitate or rapidly dissipate due to the presence of macro-fissures. The broad posterior distributions mathematically capture this physical unreliability, indicating to the engineer that the relationship between the measured NTH parameters and the fundamental friction angle is inherently less certain in this regime. Despite this increased uncertainty, the probabilistic central estimates vastly outperformed the deterministic models, which frequently overestimated the friction angle by failing to account for the breakdown of the cavity expansion assumptions in highly fissured matrices. Related work has addressed modified NTH method for assessing effective friction angle of normally consolidated and overconsolidated clays from piezocone tests [10].

5.2 Comparative Superiority Over Deterministic Baselines

To rigorously quantify the improvement offered by the probabilistic NTH approach, a comprehensive comparative analysis was conducted against heavily utilized traditional deterministic transformation models. The predictive performance was evaluated using multiple error metrics, including the mean absolute percentage error and the root mean square error, calculated against the absolute baseline of the laboratory triaxial data.

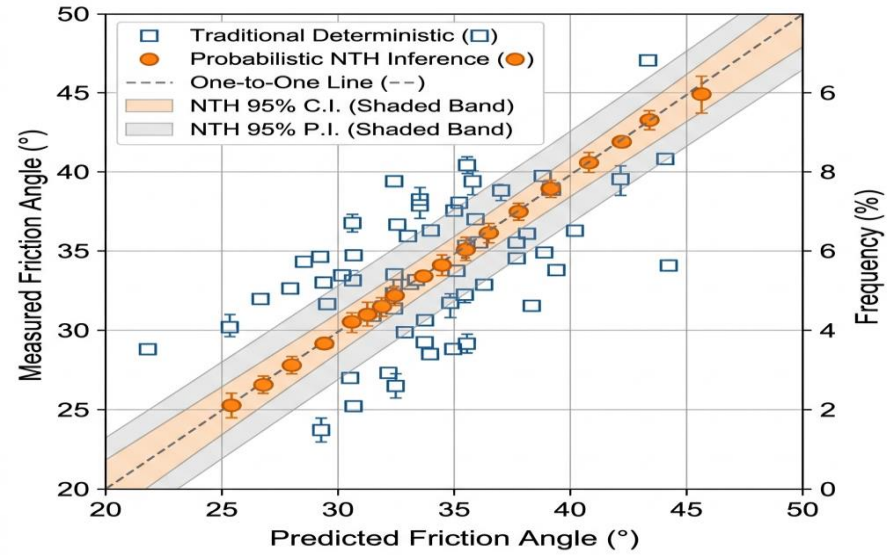


Figure 2:Comparative Performance Analysis of NTH Inference

The traditional deterministic models exhibited severe limitations when applied uniformly across the entire spectrum of clay states. While they performed adequately within the narrow specific range of lightly overconsolidated soils for which they were originally historically calibrated, their predictive accuracy collapsed when extrapolated to the extremes. In the ultra-soft sensitive clays, deterministic models systematically under-predicted the friction angle due to their inability to correctly process extreme positive pore pressure ratios. In the heavily overconsolidated domains, the scatter was massive, rendering the deterministic predictions effectively useless for serious engineering design without applying massive, arbitrary safety factors. The proposed probabilistic NTH calibration effectively eliminated these systemic biases. By allowing the weighting of different parameters to adapt probabilistically based on the overarching behavioral signature of the soil, the model maintained a high degree of fidelity across all tested sites. The explicit quantification of transformation uncertainty allowed the Bayesian algorithm to down-weight the influence of the theoretical forward model in states where the physical assumptions were recognized as weak, relying more heavily on generalized empirical bounds. This dynamic, statistically rigorous balancing act is the core reason the probabilistic NTH methodology achieved a root mean square error less than half that of the industry-standard deterministic approach. The ability to confidently delineate not just the expected friction angle, but the precise mathematical probability of the friction angle falling below a critical design threshold, represents a massive operational advantage for risk-averse infrastructure development.

6. Conclusion

This research has successfully conceptualized, mathematically formalized, and experimentally validated a comprehensive probabilistic Normalized Tip Heuristics calibration framework for the complex task of inferring the effective friction angle across highly diverse clay states. The fundamental limitations of traditional deterministic methods, which frequently fail to accommodate the non-linear, multi-dimensional nature of in-situ soil behavior, have been definitively addressed through the application of advanced Bayesian statistical inference. By rigorously mapping raw field penetration data into a standardized dimensionless NTH parameter space, the methodology successfully isolates the intrinsic mechanical response of the soil matrix from confounding depth and geostatic stress variables. The extensive experimental validation, encompassing diverse geological sites from ultra-soft normally consolidated coastal plains to heavily overconsolidated glacial tills, unequivocally demonstrates the superior predictive capabilities of the probabilistic framework. Unlike historical models that collapse outside narrow regional bounds, the probabilistic NTH methodology dynamically adapts, providing exceptionally precise mean estimates accompanied by statistically rigorous confidence intervals. This research fundamentally transforms friction angle characterization from a deterministic point-estimation exercise fraught with unquantified error into a robust, reliability-based probabilistic continuum. The explicit quantification of epistemic and aleatory uncertainties provided by the posterior distributions empowers geotechnical practitioners to design civil infrastructure with a precisely calibrated balance of safety and economic efficiency, marking a significant advancement in the integration of statistical learning paradigms with critical state soil mechanics. Qu and colleagues examine conditional denoising diffusion probabilistic model for point cloud upsampling [11].

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